

Magic numbers derived from a variable phase nuclear model

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The etymology of the term nucleus is from 1704, meaning "kernel of a nut". In 1844, Michael Faraday used the term to refer to the central point of an atom. The modern atomic meaning of the term was proposed by Ernest Rutherford in 1912, following the detection of a central massive entity by the scattering experiments of Hans Geiger and Ernest Marsden, carried out under his own supervision^[1]. To the present day, this important component of the atom has been bombarded by more energetic probes in an almost desperate attempt to reveal its internal physics. Today, almost a century after its discovery, both mechanism and phase of matter of the nucleons are still an enigma. Various models have been developed to understand selective properties of the nuclei. Some of the most popular are the shell [2], liquid drop [3], cluster [4], Moon's^[5], and double tetrahedron^[6] models, which assume a gas, liquid, semi-solid, platonic solid and tetrahedral solid phase for the nuclei respectively. It is interesting to note that each of these models is able to describe very successfully certain selected properties of the nuclei; however, none of them is able to give a comprehensive description. Most of the characteristics of the different phases are mutually exclusive. I shall here introduce the Variable Phase Model of the nucleus [2], based on the projection of a hypertetrahedral nuclear structure into our view of perception, which is limited to the observation of three dimensions. In this model, the phase of the nucleus varies between the various phases of matter according to the angle of projection of the hyper dimensional nuclear entity, thus redefining mass as a physical parameter having both real and imaginary components.

Discovery of the magic numbers

"Magic numbers" were first discovered by physicist Maria Goeppert-Mayer^[8]. Careful observation of the nuclear properties of elements showed certain patterns that seemed to change abruptly at specific key elements. Mayer noticed that magic numbers applied whether one counts the number of neutrons (N), the atomic number (Z), or the sum of the two, known as mass number (A). Examples are Helium Z=2, Lead Z=82, Helium N=2, Oxygen N=8, Lead N=126, Neon A=20, Silicon A=28. Magic numbers in the nuclear structure have been coming up during all this time, but no plausible explanation for their existence has ever been given. Interestingly, there are peaks and dips for binding energy, repeating every fourth nucleon. This periodicity is one clear indication of a geometrical structure within the nucleus. In particular,

those nuclei that can be thought of as containing an exact number of alpha particles (2P+2N), are more tightly bound than their neighbours. This effect is more pronounced for the lightest

nuclei, but is still perceptible up to A = 28. For those nuclei with A > 20, the number of neutrons exceeds the number of protons, so some sort of distortion occurs within the cluster, as we shall discuss.

It is found that nuclei with even numbers of protons and neutrons are more stable than those with odd numbers. This comes from the fact that the physical structure must have an even number of vertices. A type of regular polyhedron would satisfy this condition, since no regular polyhedron exists with an odd number of vertices. These specific "magic numbers" of neutrons or protons which seem to be particularly favoured in terms of nuclear stability are:

2, 8, 20, 28, 50, 82, 126

Note that the structure must apply to both protons and neutrons individually, so that we can speak of "magic nuclei" where any one nucleon type, or their sum, is at a magic number.

We find magic numbers in the elements most abundant in nature:

2H1 (Hydrogen, 1st most abundant - 74% of the universe) 1+1=2= magic

4He2 (Helium, 2nd most abundant – 24% of the universe) 2,2 = both magic

1608 (Oxygen 3rd most abundant – 1% of the universe) 8.8 = both magic

Maria Goeppert-Mayer proposed that magic numbers should be explained in the same way as the electron shell model applies to electrons. So, in such a nuclear shell model, when a nuclear shell is full and the structure is formed (equivalent to saying that the nucleons have used up all of the possible sets of quantum number assignments), a nucleus of unusual stability forms. This concept is similar to that found in an atom where a filled set of electron quantum numbers results in an atom with unusual stability, usually an inert gas. When all the protons or neutrons in a nucleus are in filled shells, the number of protons or neutrons is called a "magic number." Visualizing the densely packed nucleus in terms of orbits and shells seems much less plausible than the corresponding shell model for atomic electrons. You can possibly accept the fact that an electron can complete many orbits without running into anything, but you would expect protons and neutrons in a nucleus to be in a continuous process of collision with each other. Despite the expectations, dense-gas models of nuclei with multiple collisions between particles didn't fit the data, and remarkable patterns like the "magic numbers" in the stability of nuclei suggested the seemingly improbable shell structure, which defines nuclei in layers similar to those of the electron shell model (Fig.1). In our model, a shell is built up of a structured layer.

Quoting Maria Mayer in "The shell model": "One of the main nuclear features which led to the development of the shell structure is the existence of what are usually called the magic numbers. That such numbers exist was first remarked by Elsasser in 1933. What makes a number magic is that a configuration of a magic number of neutrons, or of protons, is unusually stable whatever the associated number of the other nucleons. When Teller and I worked on a paper on the origin of the elements, I stumbled over the magic number. We found that there were a few nuclei which have a greater isotopic, as well as cosmic, abundance than our theory, or any other reasonable continuum theory, could possibly explain. Then I found that those nuclei had something in common: they either had 82 neutrons, whatever the associated proton number, or 50 neutrons. Eighty-two and fifty are magic numbers. That nuclei of this type are unusually abundant indicates that the excess stability must have played a part in the process of the creation of the elements..."

Fig.1 Relation between quantum shells and magic numbers

Doubly Magic nuclei

Doubly magic nuclei, are those nuclei which have both neutron number and proton number equal to a magic number. All such nuclei are particularly highly stable and are called "doubly magic".

$${}^{4}_{2}\text{He} \ {}^{16}_{8}0 \ {}^{40}_{20}\text{Ca} \ {}^{48}_{20}\text{Ca} \ {}^{208}_{82}\text{Pb}$$

Lead-208 is one such example of a "doubly magic" nucleus as it has both 82 protons and 126 neutrons. Calcium it yet another example of this exceptional stability quality since it has two of them. The existence of several stable isotopes of calcium is due to its magic proton count of 20. The two highlighted isotopes have neutron numbers 20 and 28 and a proton count of 20, all

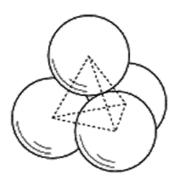
magic numbers. Atomic nuclei consisting of such a magic number of nucleons have a higher average binding energy than that calculated from the semi-empirical Weizsaecker formula, and also anomalously low masses and high natural abundances.

The existence of these magic numbers suggests some special shell configurations, like the electron shells in the atomic structure. They represent one line of reasoning which led to the development of a shell model of the nucleus. Other forms of evidence suggesting a geometrical structure include the following ^[9]:

- Enhanced abundance of those elements for which Z or N is a magic number.
- The stable elements at the end of the naturally occurring radioactive series all have a "magic number" of neutrons or protons.
- The neutron absorption cross-sections for isotopes where N = magic number are much lower than surrounding isotopes.
- The binding energy for the last neutron is a maximum for a magic neutron number and drops sharply for the next neutron added.
- Electric quadrupole moments are near zero for magic number nuclei.
- The excitation energy from the ground nuclear state to the first excited state is greater for closed shells.

Tetrahedral stacking

Following the interpretation of Maria Mayer, the striking evidence for a structure in the nucleus was surprising at first, because common sense tells us that, a dense collection of strongly interacting particles should be bumping into each other all the time, resulting in redirection and perhaps loss of energy for the particles. This idea at first seems to violate Pauli's exclusion principle, but it does not. Keep in mind that the exclusion principle itself has been devised in the first place due to the lack of information about the geometrical structure of the electron shells.



Tetrahedral stacking

Fig 2.

Let us have a closer look at the nucleus. Assuming some sort of close packing arrangement for the nucleons, and assuming that the nucleus is perceived as a three dimensional object, it can be shown that the least number of nodes a three dimensional structure can have is equal to four, and that the simplest stable stacking structure is that of a tetrahedron (Fig.2). Four nucleons are thus required to fill the nodes of a tetrahedron. A nucleus with two protons and two neutrons would thus satisfy the most basic stable three dimensional structure, which would in fact give the nuclear structure of Helium 4 – otherwise known as the alpha particle.

Calculations of potential model, constrained by the hadrons spectrum for the confinement of the relativistic quark^[10] and coloured quark exchange model^[11], are also consistent with a tetrahedron formation of the nucleus. Also note that this tetrahedron structure is not something new, but dates back to 1964 in the work of Linus Pauling.^[12] Once all four nodes of a tetrahedron are occupied, the next nucleon cannot be permitted at the same energy level, and the next spherical standing wave^[13] (spherical tetrahedron) has to start forming. Recently, tetrahedron structures have regained interest in the study of the nuclear structure.^[6] This theory also explains the emission of alpha particles from the nucleus. Many radio-nuclides achieve increased nuclear stability by emitting an alpha particle (a tetrahedral structure) rather than a single proton or neutron. This suggests that many isotopes contain one or more alpha particles in their nucleus.

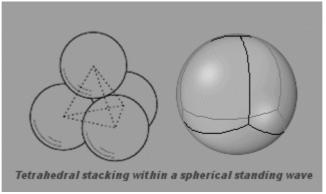


Fig 3.

If you consider every particle, being it a proton, neutron, or electron, as being restricted in space relative to its neighbour particle by the shape of a particular structure (spherical standing wave), there would be no need for any principle to show how and why these particles can never collide with each other, since they would be seating within the nodes of the frame. We know that collisions between electrons or nucleons in a particle are very infrequent. As I shall discuss later on, the possibility that matter exists in higher dimensions than three must not be excluded, in which case, two distinct three dimensional objects can overlap within the same three dimensional space without colliding. In fact from my own space time conversion system [14] one can easily deduce that mass is a three dimensional structure of energy, that is a six dimensional unit, and not a structure of point particles.

Note in the above diagram (Fig.3) that each nucleon is itself a spherical tetrahedron, each of which has three of its nodes touching the neighbouring nucleons, and one touching the external spherical tetrahedron of the formation. Thus the nodes of the spherical tetrahedral standing wave, are formed at the points where the stacked nucleons touch the external sphere. In a three dimensional nucleus, a tetrahedral stack consisting of four nucleons, such as an alpha particle, can accommodate an infinite number of stacked layers, while keeping the same properties of a tetrahedron. For example if six other nucleons are added at the base of a four

nucleon tetrahedral stack, a new tetrahedral structure consisting of ten nucleons will be formed, having its three outermost nucleons touching a bigger external spherical tetrahedron.

What does an incomplete tetrahedron standing wave look like

Up to now we have considered filled up shells which equate to a complete spherical tetrahedron, equivalent to the Helium-4 nucleus (Fig.3). But what does the standing wave look like when the number of nodes is not equal to the number of vertices, for example in a Hydrogen nucleus? Does this result in a non-uniform structure within the spherical standing wave? How can a spherical wave contain one, two or three nodes? In the analogous situation of the electron shell model, chemists just put up the next shell and place the extra electrons orbiting around, but this picture is totally wrong.

Let us assume that a complete shell structure, say shell 's' has just been completed and a spherical tetrahedron has formed. We know that three similar spherical standing waves (known as subshell p) are now required until we have enough nodes to create a bigger tetrahedron shell. An analogous example to this process is with the case of using an oscilloscope trace while increasing the input frequency. If the wave signal frequency at its input matches exactly its time base frequency f, then, a single wave will appear at a standstill on the screen. If we double the input frequency to $2\boldsymbol{f}$, we see two standstill waves on the screen. But what do we see in between the two frequencies? If one increases the frequency slowly from f to 2f, the result is that the single wave will start moving across the screen until it slows down again and 'morphs' into the shape of two waves. This is exactly what happens at the nuclear level as extra nodes are added. This movement of the standing wave results in ROTATION or spin of the structure, until it builds up further nodes and 'slows down' into the new zero spin complete shells. Another analogy, this time mechanical, is that of a two spoke wheel being viewed under the light of a stroboscope. Slowing down the speed of the wheel will result in the observation of a stationary two, four, six, eight ... spoke wheel. For revolutions which are not exact integer fractions of the stroboscope frequency, the spokes will appear to be either in random positions, or rotating and the observer cannot determine the actual number of spokes.

All incomplete, spherical, 3d standing waves have non-zero spin values and are pictured as various standing wave components. Zero spin and complete shells are achieved at the same time for the same reason - the number of nodes are equal to the number of nodes of a spherical platonic. Hydrogen(2) has a non zero spin since it has 1/2 the nodes required to complete the smallest and simplest spherical tetrahedron, which is achieved by a minimum of four nodes equivalent to the alpha particle, which in fact does have a zero spin.

For many years, physicists have known that energy particles spin as they travel. For example, electrons appear to be continually making sharp 180-degree turns or half spins as they move through the atom. Quarks are often seen to make one thirds and two thirds spins when they travel. No one in the mainstream has provided a truly adequate explanation as to why this is happening. In our model, spin is just the movement of the spherical standing wave whenever the structure of nodes does not 'lend itself' to create a complete three dimensional structure.

What's wrong with the standard model

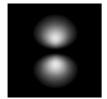
The conventional 'bunch of grapes' nuclear structure (Fig.4) described in the standard model ^[15] has big flaws, and lacks to predict too many well known characteristics of the nucleus. One of its biggest flaws, is that it regards all elementary particles as point particles (zero dimension), which results in infinite energy when electric field energy is taken into account. However, it does give us a vague clue that multiple spherical standing waves can combine into another bigger entity, while maintaining themselves as separate identifiable similar structures. Evidence for this clustered structure comes from electron and alpha particle emissions from the atom.

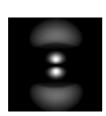


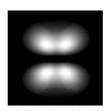
Bunch of grapes conventional nucleus model gives no hint to nuclear magic numbers or shell structure.

However due to the lack of a defined structure, the conventional 'bunch of grapes' model tends to be an overly complicated model. In such a model, there is the need for the nuclear binding energy to overcome the tendency of the nuclear components to fly apart, because of the mutual repulsion of the positive proton charges. The mass deficiency of atomic nuclei has been hypothesized as the cause of this nuclear binding, and for this to take place, a new type of strong force has been hypothesized to exist in the form of 'exchange forces' between nuclear components. To complicate the issue further,

a new class of particles like the meson have been invented to account for the force exchange mechanism. This hypothesis has in fact never been proved. The standard model also gives no hint to the existence of the nuclear magic numbers and no way to predict, or even explain the existence of its obvious shell structure. Since Bohr's orbiting electron model failed to describe the actual orbital distribution of the electron cloud, it had been concluded that the electrons motion is not governed by any ordered motion or structure, but is completely random. From that time, even if today we no longer learn about 'orbiting electrons', Heisenberg principle has unfortunately been established into mainstream science. The interpretation of such principle is that the atomic structure and the interactions of its electrons are random and can be discussed only statistically as probabilistic distributions of a random motion. On the other hand, we have nature, that shows us otherwise – crystal lattices^[16] do not build up in random shapes, but in very specific shapes like simple cubic, body centred cubic, cubic close packed etc.. Unfortunately, to the present day, science gave up the search for a 'physical' model and most people prefer to ignore hard evidence in favour of an outdated principle, referring to electromagnetic radiation patterns (Fig. 5) as probabilistic distribution of electron 'clouds'.







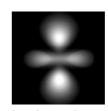




Fig. 5 – Some 'electron probability density' plots for the Hydrogen atom

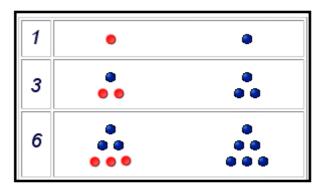
One can solve the binding force problem, by showing that the most stable nuclei, are tightly bound due to being in the stable nodal positions of a spherical standing wave. Before going into the actual hyper geometric structure model of the nucleus, we shall first hack the magic number sequence into a predictable three dimensional configuration.

Deriving the Magic Number sequence using a physical model

The first attempts to hack, or reverse engineer, the Magic number sequence into a geometric progression by the use of a physical model, date back to 1964 during Pauling's research^[12]. From his records we can see that today's magic numbers are exactly the same as those listed under his 'Observed' values. Unfortunately it seems that Pauling never got a solution to generate the correct sequence.

An interesting characteristic of the collection of protons and neutrons is that a nucleus of odd mass number A will have a half-integer spin and a nucleus of even A will have an integer spin. This highly suggests that the real structure is based on pairing of nucleons. The suggestion that angular momenta of nucleons tend to form pairs is supported by the fact that all nuclei with even Z and even N have nuclear spin I=0. In this section, we will need to stack pairs of tetrahedrons in our nuclear structure, so I shall first cite the triangular and tetrahedron number sequences. [17.18] These are very simple numerical sequences which are very useful for this study.

Triangular Numbers



A triangular number is a number that can be arranged in the shape of an equilateral triangle. The n^{th} triangular number is the sum of the first n numbers added up, for example 0+1=1, 0+1+2=3, 0+1+2+3=6, 0+1+2+3+4=10, etc.. The sequence of triangular numbers is:

The formula for the nth triangular number is given by

$$TRI(n) = (n/2)(n+1)$$

Square Numbers - relation to triangle numbers

An important relation we will find useful, between square and triangular numbers is:

$$SQR(n) = TRI(n-1) + TRI(n) = n^2$$

Tetrahedral Numbers

Analogously, a tetrahedral number, is a number that can be arranged like a tetrahedron.

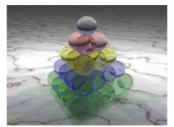


Fig.6 - 3D view of tetrahedral stack

The n^{th} tetrahedral number is the sum of the first n triangular numbers added up, for example 0+1=1, 0+1+3=4, 0+1+3+6=10, etc.. The sequence of tetrahedral numbers is thus:

The formula for the nth tetrahedral number is given by

$$TETRA(n) = (n/6)(n + 1)(n+2)$$

Hacking the Magic elements structure

Now let's see how these two number series apply to a 3D tetrahedral pack of nucleons. Level n=1 is the top level - the black sphere, level n=2 is the red, n=3 is the yellow and so on... The first equation for the triangular numbers at n=3 gives us the number TRI(3)=6. So, 6 is the number of spheres in the yellow layer. Putting n=3 in the tetrahedral equation, gives us the number TETRA(3)=10, and that is equal to the total of spheres making up the tetrahedron whose base is formed by the yellow spheres, that is, the total of yellow, red and black spheres. If one wants to find the number of spheres required for the whole 5-layer tetrahedron, it will be the fifth number in the tetrahedron number series, or TETRA(5), that is 35. If you subtract the 4th triangular number from 35, you get the number of spheres of the whole tetrahedron, less the blue slice, which will be TETRA(5)-TRI(4)=35-10=25.

Next we will apply two other facts already mentioned to hack the magic number structure. Fact no.1 is that nuclei up to magic number Z=20 have equal numbers of protons and neutrons, whilst for Z>20, the balance is lost. Fact no.2 is that from the binding energy graph of elements, from which it is clear that the nuclei build up as a **double structure**^[19]. From fact no.1 we deduce that up to the formation of the structure containing 20 protons, the nucleus will look as a perfectly symmetrical structure, in electromagnetic terms, a perfect dipole. Actually adding a similar structure for neutrons, it will look as a perfect quadrupole. From fact no.2, we deduce that each tetrahedron level, builds up in pairs, and the next level tetrahedron is not started unless both pairs of the previous stage have been completed.

Double tetrahedron nuclear structure



Z=2 (n=1) Z=8 (n=2) Z=20 (n=3)

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Fig 7.

Applying these rules we can now start from level n=1 until we get to Z=20 at n=3.

For n=1, TETRA(1)=1, so the double tetra structure closes at 1x2 = 2

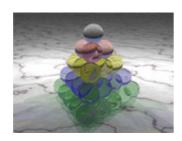
For n=2, TETRA(2)=4, so the double tetra structure closes at 4x2 = 8

For n=3, TETRA(3)=10, so the double tetra structure closes at 10x2 = 20

You may have already noticed, that these are the first 3 of the magic number series

2, **8**, **20**, 28, 50, 82, 126

Now that we are at Z=20 (n=3), we know that the geometrical symmetry from here on is lost, that is, the double structure (or 3 dimensional dipole) will no longer act as two perfect tetrahedron structures, and so, we can no longer assume a simple symmetric double tetrahedral structure. However, things do not complicate much, because what happens from here is that the two tetrahedron pairs now hinge together or share the same space, much like covalent bonds are known to share the same orbits in chemistry. In electromagnetic terms, the dipole will no longer act as a purely resistive electromagnetic standing wave, but become slightly reactive due to their overlap.





Double tetrahedron nuclear structure n=5 gives magic number structure Z=50

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Fig. 8

For example, if we take the above 35 nucleon tetrahedron (n=5) shown in Fig.8 (left), its companion tetra will be underneath it, in inverted position, with its green layer occupying the same space previously occupied by the blue layer (triangular level 4=n-1) of the top tetrahedron (Fig.8 right). The green layer of the latter, will also occupy the same space previously occupied by the blue layer of the inverted tetra. Thus, the total number of nucleons for the magic dual structure for n=5, would be given by Magic(n) = 2*[TETRA(n)-TRI(n-1)] = 2*(35-10) = 50. This simple rule is observed for all magic nuclei having Z>20, that is for n>3. The table below shows how these numbers can be easily worked out by subtracting the respective triangular hinge layer (n-1) from each tetrahedron number, and multiply the result by two to get the number of nucleons of the complete double structure. The bonding layer, as chemists would call it, is always one layer above the base of the tetrahedron.

| Level n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|----|----|----|----|-----|-----|
| TETRA (n) | 1 | 4 | 10 | 20 | 35 | 56 | 84 | 120 |
| TRI (n-1) | - | - | - | 6 | 10 | 15 | 21 | 28 |
| Total nucleons per double structure Magic(n) (Magic number/closed shell Z sequence) For 3 ≥n≥ 1, Z(n) = 2*TETRA(n) For n>3, Z(n)= 2*[TETRA(n)-TRI(n-1)] | 2 | 8 | 20 | 28 | 50 | 82 | 126 | 184 |

Table 1

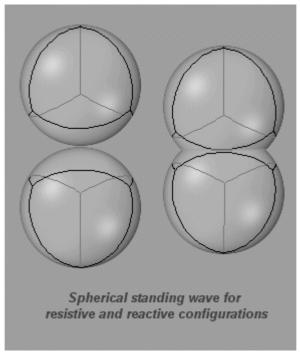


Fig 9. - Resistive (left) & Reactive (right) dipole configurations

The lowest values 2, 8, and 20 agree with independent nucleon motion into a single particle potential, like a harmonic oscillator. Nucleons' stacking positions occur at the vertices of the two spherical standing waves or the resistive configuration, shown above in Fig.9. Mathematically the sequence of complete shell nucleon numbers for Z<=20 is given by the same equation that gives the total number of spheres of two symmetrical tetrahedron stacks forming an ideal 3D dipole:

For $3 \ge n \ge 1$: Z = Magic(n) = 2*TETRA(n) = (n/3)(n+1)(n+2)

For
$$3 \ge n \ge 1$$
: Magic(n) = $(n/3)(n^2+3n+2)$

giving series: 2, 8, 20

The magic numbers 28, 50, 82, and 126 agree to those nuclei with a strong spin-orbit coupling (by Maria Mayer and Jensen) which have the mentioned nuclear co-valent bond type structure as discussed above. Nucleon stacking positions occur at the vertices of the two spherical standing waves shown on the right, or the reactive configuration of the above figure. So, for Z>20, the total number of nucleons is given by the above equation, less a pair of two triangular layers which represent the binding energy (or nuclear anomalous mass deficiency). In electromagnetic terms, this binding energy or missing mass is due to the reactive component of the capacitive dipole, a situation analogous to the real and apparent electric power in reactive loads we learn in electrical theory.

For n>3: Magic(n) = 2*TETRA(n) - 2*TRI(n-1) = (n/3)(n+1)(n+2) - n(n-1)

For
$$n>3$$
: Magic(n) = $(n/3)(n^2+5)$

giving series: 28, 50, 82, 126, 184, ...

We have therefore finally hacked the magic number sequence into a physical model based on the double tetrahedron structure and simple dipole structure of the nucleus as proposed here. Not only do the above two geometrical sequences derive all known magic numbers (2, 8, 20, 28, 50, 82, 126) but also derive the magic number 184, which is predicted by many scientists to be the next higher magic number. I have also shown how today's classical 'bunch of grapes' nuclear model can be geometrically arranged for the magic nucleons case, while giving a plausible explanation for the missing mass that is normally attributed to binding energy. Binding energy has been replaced by the electromagnetic energy which holds the nucleons fixed upon the nodes of a spherical standing wave.

Deriving the Quantum theory nucleon and electron shell capacity

If we slice a stacked tetrahedron in two level layers, we find out that the total number of nucleons agrees with that predicted from Quantum Theory for each quantum shell, assuming that the number of protons and neutrons are equal.

| Level n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--|---|------|---|------|----|------|----|------|
| Shell number n | | 1(K) | | 2(L) | | 3(M) | | 4(N) |
| TRI(n) | 1 | 3 | 6 | 10 | 15 | 21 | 28 | 36 |
| Nucleons in Quantum Shell = Nucleons in two level slice = TRI (2 n) + TRI(2 n -1) | | 4 | | 16 | | 36 | | 64 |
| For the case Protons= ½*nucleons | | 2 | | 8 | | 18 | | 32 |

Table 2

If we define each principal quantum shell number \mathbf{n} , as a two level slice of a tetrahedron, then, the number of nucleons per slice = TRI(n) + TRI(n-1) where $n=2\mathbf{n}$. As already mentioned, the sum of two consecutive triangular numbers is always the square number of the highest triangular level.

The total number of protons or electrons in such a slice = $\frac{1}{2}$ * total nucleons in slice

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So, Z_{max} = \frac{1}{2} [TRI (n ) + TRI (n-1) ] 

Z_{max} = \frac{1}{2} n<sup>2</sup> 

Z_{max} = \frac{1}{2} [ 4\boldsymbol{n}^2 ] 

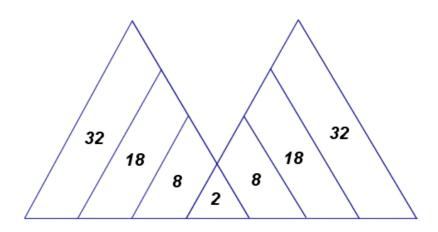
Z_{max} = 2\boldsymbol{n}^2, giving the sequence 2,8,18,32,.. for principal quantum numbers 1,2,3,4...
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This is the well known empirical formula which defines the maximum number of electrons in the set of orbitals, also known by their spectroscopic designation K, L, M, N, etc.). We now reconfirm, that each quantum shell, is made up of two sub shells, or a two layer slice of a tetrahedral structure, which is composed of the triangular levels TRI(2n) and TRI(2n-1). It thus follows, that the electron structure is closely related or simply a direct effect of the nuclear structure. This is the same as saying that the far field radiation pattern of a radio antenna, can be directly deduced by knowing either its near field pattern, or its dipole structure^[20].

Nuclear structure of inert gases

Two independent tetrahedron stacks will each generate Z_{max} sequence 2, 8, 18, 32 A tetrahedron stack pair would generate Z_{max} sequence 2, 2, 8, 8, 18, 18, 32, 32 A simplex stack pair having their top simplex projected over the same space in 3D, will thus generate Z_{max} sequence 2, 8, 8, 18, 18, 32, 32 and be like two tetrahedral stack structures with their uppermost tetrahedrons overlapping the same space. It will also be equivalent to a conventional electron shell structure: s, sp, spd, spd, spdf, spdf.

Building up the dual overlapping stack, gives the element sequence 2, 2+8, 2+8+8, 2+8+8+18, 2+8+8+18+18, 2+8+8+18+18+18+32, 2+8+8+18+18+32+32 which results in elements 2, 10, 18, 36, 54, 86, 118 – well known as the inert gases (including the recently discovered Unonoctium - element 118!). Again, the sharing of the top tetrahedron indicates higher dimensional entities. Taking into account both protons and neutrons, the structure will look like four tetrahedral stacks, sharing a common central tetrahedron. For the sake of clarity, figure 10 shows only two of the tetrahedral stacks, the other two would be stacked over the other two faces of the central tetrahedron.



Inert elements proton structure
Central tetrahedron is shared between the two stacks

Fig. 10

This physical model also unveils a property of mass which is not yet conceived in today's science literature - the reactive or imaginary component of mass. As is easily observed above, all magic number structures can be understood in terms of electromagnetic energy components, and since such components are complex in nature (complex means they have both real and imaginary parts), a structure which can be completely described in such terms will have the same properties of its constituents. Looking at the periodic table of elements, one should not only visualise nucleons simply bunching up as tetrahedral stacks of hard steel balls, but (hyper)tetrahedral stacks of spherical electromagnetic standing waves with varying real and imaginary energy components. Spin and angular momentum are direct effects of the imaginary components. Only when we take these components into account, can we start to appreciate, predict and master the properties of the elements.

Tetrahedral or Hypertetrahedral stacking?

The structure for the pair of tetrahedrons for Z>20, shown in Fig.8, leads one to consider a hypertetrahedron (simplex or higher dimension) type of stacking, in favour of the simple tetrahedron stacking. To understand why, let's analyse a simpler situation with a pair of three dimensional spheres, as shown below in Figure 11:

Understanding dimensional projection

3D view 2D projection

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Fig. 11

You can see two different cases, showing the 3D spheres at a different angle of projection onto a 2D plane, one at say, time=t, and another at time $t=t+\Delta t$. The relation of their position with respect to the 2D projection plane is described by the projection or phase angle. In real life we can see the 3D view with no problem. If one takes photos from underneath them however, or simply projects their shadows on a screen, the result would be that shown in the 2D projection. Now, if one has got only a photo of the 2D projection, it will be confusing, since it will appear that two individual circles (lower left) are merging into two overlapping circles (lower right). The surface area of the two circles on the right is less than that of two separate circles, and can diminish to the surface area of a single circle, at which point the observer cannot even know whether there is one or two circles. The observer will call the effect as missing area, and if density (in a 2D world like Flatland) is defined as mass per unit area, the effect would be that of a missing mass.

Now, you do not really have to imagine what a 4D hyper sphere would look like (it is impossible, even if you try), but just apply what you understood for the lower dimensional case, to the case, where the spheres exist in 4D, and the projection is on a 3D kind of screen. Two hyper spheres separated at a distance greater than their diameter, would look like two independent 3D spheres, and the image would look like it is the real thing. But when their horizontal distance is decreased, or the phase angle increases, strange things start to happen. They seem to be overlapping into the same space, until you can see only one sphere! One would actually see the two separate masses combining into a single mass, sharing the same

space until their mass becomes that of a single sphere. The observer will call the effect of the missing volume as missing mass - a 'mass defect', as Einstein called it, accompanied with a respective increase in internal or binding energy. In the macro world, the effect will show as a decrease in density, or change of phase of matter from solid to liquid to gas to plasma and finally to vacuum, accompanied with a respective increase in internal energy. As you can surely understand, when such effect is observed, it is a clear indication, that your image is not the complete picture, and that the real thing is operating at a higher dimensional level. One cannot really know if it's just one or many levels up, but it is certain that the observed dimension is a limited projection of the entity being observed. And that's exactly what's happening with our tetrahedral stacking within the nucleus. For Z<20, we see pairs of 3D tetrahedral stacking operating as normal 3D stacks would do. Only as Z increases further, we realize that strange things start to happen, the pair of tetrahedral stacks overlap and partly share the same space, resulting in a mass defect. At that stage, it becomes clear that a tetrahedral nuclear model is just a limited projection in 3D, of a higher dimensional entity - a hypertetrahedral stack. You will now clearly understand, that if we define mass as a 3D entity, it will always be a shadow effect. In other words, the term mass refers to the apparent component of a much complex entity. This is the reason of the long time failure of generating a 3D physical model to represent the complete list of known elements. The variable phase angle is the reason for which a nuclear model cannot assume any single specific phase of matter. Similarly to electronic components, where in practice, no component can be assumed purely resistive or purely reactive, the phase of matter and hence of the nucleus, cannot be considered to be either purely solid, or purely vacuum energy, both of which are purely theoretical limits. Such model must be considered as a hyper dimensional structure, otherwise, all those elements which do not happen to project into a simple enough 3D structure like the case for magic numbered nucleons and inert gases, will have spin, which will result in a rotating/overlapping three dimensional projection of hypertetrahedral stacks popping in and out of the 3D 'screen'.

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